

# Geometric construction and kinematic analysis of a 6R single-loop overconstrained spatial mechanism that has three pairs of revolute joints with intersecting joint axes

**Citation for published version:**

Kong, X 2016, 'Geometric construction and kinematic analysis of a 6R single-loop overconstrained spatial mechanism that has three pairs of revolute joints with intersecting joint axes', *Mechanism and Machine Theory*, vol. 102, pp. 196-202. <https://doi.org/10.1016/j.mechmachtheory.2016.04.002>

**Digital Object Identifier (DOI):**

[10.1016/j.mechmachtheory.2016.04.002](https://doi.org/10.1016/j.mechmachtheory.2016.04.002)

**Link:**

[Link to publication record in Heriot-Watt Research Portal](#)

**Document Version:**

Peer reviewed version

**Published In:**

Mechanism and Machine Theory

**Publisher Rights Statement:**

© 2016 The Authors. Published by Elsevier Ltd. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>).

**General rights**

Copyright for the publications made accessible via Heriot-Watt Research Portal is retained by the author(s) and / or other copyright owners and it is a condition of accessing these publications that users recognise and abide by the legal requirements associated with these rights.

**Take down policy**

Heriot-Watt University has made every reasonable effort to ensure that the content in Heriot-Watt Research Portal complies with UK legislation. If you believe that the public display of this file breaches copyright please contact [open.access@hw.ac.uk](mailto:open.access@hw.ac.uk) providing details, and we will remove access to the work immediately and investigate your claim.

# Geometric Construction and Kinematic Analysis of a 6R Single-loop Overconstrained Spatial Mechanism That Has Three Pairs of Revolute Joints with Intersecting Joint Axes

Xianwen Kong

*School of Engineering and Physical Sciences, Heriot-Watt University,  
Edinburgh, UK, EH14 4AS Tel: +44(0)1314513688*

---

## Abstract

This paper deals with the geometric construction and kinematic analysis of a single degree-of-freedom 6R overconstrained spatial mechanism (6R mechanism for short) that has three pairs of revolute (R) joints with intersecting joint axes. How to construct a 6R mechanism with three pairs of R joints with intersecting joint axes is presented first. The kinematic analysis of the 6R mechanism is then discussed. The analysis shows that the 6R mechanism usually has two solutions to the kinematic analysis for a given input. In two configurations in each circuit of the 6R mechanism, the axes of four R joints are coplanar, and the axes of the other two R joints are perpendicular to the plane defined by the above four R joints.

*Keywords:* Overconstrained Mechanism, 6R Mechanism, Kinematic Analysis, Geometric Construction Approach

---

## 1. Introduction

The research on single degree-of-freedom (DOF) single-loop overconstrained mechanisms started more than one and half centuries ago. So far, a number of single-loop overconstrained spatial mechanisms [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24] have been proposed.

---

*Email address:* X.Kong@hw.ac.uk (Xianwen Kong)

Meanwhile, different approaches, such as geometric methods [1, 2, 24], construction approaches [3, 4, 8, 14, 22], algebraic approaches [6, 12, 15, 16, 17, 18, 19, 20, 21, 23] and numerical methods [7], have been developed for obtaining 6R mechanisms. For a comprehensive list of 6R mechanisms identified before 2009, refer to [15]. A historic review of research on 6R mechanisms by Russian researchers can be found in [25]. In addition, an efficient method was proposed for calculating the DOF of an overconstrained mechanism in [26]. Despite the advances in the past decades, searching for 6R mechanisms is still an open issue.

Although the successful industrial applications of single-loop overconstrained mechanisms are not many so far, single-loop overconstrained mechanisms have been used in the development of deployable structures [14] and parallel mechanisms [27, 28]. The application of 6R mechanisms in mobile robots [29] and other devices [25, 30] is being explored.

Recently, several 6R mechanisms, which have two pairs of adjacent R joints with parallel joint axes and one pair of non-adjacent R joints with parallel joint axes, have been systematically obtained using a dual-quaternion based approach [19, 20, 21] or a construction approach [22, 24]. Before the systematic type synthesis of this class of 6R mechanisms, only two such 6R mechanisms were proposed in the literature and both were unfortunately ignored by researchers working on overconstrained 6R mechanisms for decades [11, 18, 19, 22]. The first one was proposed by Mudrov in 1976, as pointed out in [25], and can be regarded as a special case of the double-Bennett hybrid 6R mechanism [4]. The second one was obtained during the investigation of novel steering techniques in reference [9]. Following the success of type synthesis of this class of 6R mechanisms, it is logic to obtain 6R mechanisms that have three pairs of R joints with intersecting joint axes, including two pairs of adjacent R joints with intersecting joint axes and one pair of non-adjacent R joints with intersecting joint axes. It is apparent that two 6R mechanisms that have three pairs of R joints with intersecting joint axes can be obtained from plane symmetric Bricard mechanism and line symmetric Bricard mechanism by imposing certain geometric constraints, like in the generation of 6R mechanisms that have three pairs of R joints with parallel joint axes [22]. Whether there are other classes of 6R mechanisms that have three pairs of R joints with intersecting joint axes has not been investigated.

Inspired by the geometric construction of Bricard 6R mechanisms [1, 2], especially the trihedral Bricard mechanism and the Type III Bricard's flexible Octahedron, this paper aims at revealing a 6R mechanism that has three

pairs of R joints with intersecting joint axes using a geometric construction approach.

This paper is organized as follows. The geometric construction of a 6R mechanism that has three pairs of R joints with intersecting joint axes will be given in Section 2. The kinematic analysis of the 6R mechanism will be presented in Section 3. In Section 4, the characteristics of the 6R mechanism will be revealed. Finally, conclusions will be drawn.

For simplicity reasons,  $\sin \theta_i$  and  $\cos \theta_i$  are denoted by  $S\theta_i$  and  $C\theta_i$ , respectively.

## 2. Description of a 6R mechanism that has three pairs of R joints with intersecting joint axes

In this section, the geometric construction of a 6R mechanism is presented first. The D-H link parameters of the 6R mechanism are then given.

### 2.1. Geometric construction of a 6R mechanism

A 6R mechanism that has three pairs of R joints with intersecting joint axes can be constructed as follows (Fig. 1):

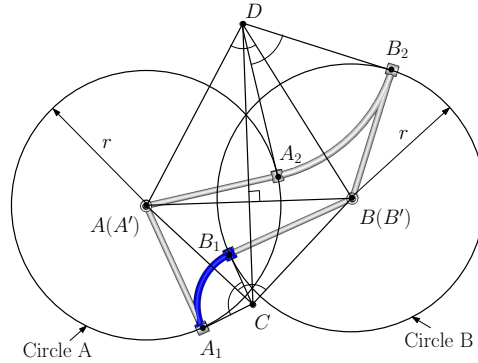


Figure 1: Construction of a 6R mechanism.

Step 1: Draw six lines for placing R joints.

At first, draw a kite ACBD where  $|AC| = |BC|$  and  $|AD| = |BD|$  as well as two circles of radius  $r$  ( $r \leq |AC| = |BC|$  and  $r \leq |AD| = |BD|$ ) with their centers at  $A$  and  $B$  respectively. Then draw:

- Lines  $AA'$  and  $BB'$  that are perpendicular to the kite ACBD.

- Lines  $CA_1$  and  $CB_1$  that are tangent to circles  $A$  and  $B$  respectively at points  $A_1$  and  $B_1$  such that  $\angle ACA_1 = \angle BCB_1$ .
- Lines  $DA_2$  and  $DB_2$  that are tangent to circles  $A$  and  $B$  respectively at points  $A_2$  and  $B_2$  such that  $\angle ADA_2 = \angle BDB_2$ .

Lines  $AA'$ ,  $BB'$ ,  $CA_1$ ,  $CB_1$ ,  $DA_2$  and  $DB_2$  are the six lines required for placing six R joints.

Step 2: Construct a 6R mechanism using the six lines obtained in Step 1.

Place six R joints 1, 2,  $\dots$  6 along lines  $CA_1$ ,  $AA'$ ,  $DA_2$ ,  $DB_2$ ,  $BB'$ , and  $CB_1$  respectively and connect them in the sequence of 1-2-3-4-5-6-1. One then obtains a 6R mechanism that has three pairs of R joints with intersecting joint axes 1-2-3-4-5-6-1. Here two adjacent R joints are connected using a circular link if the two joint axes intersect with each other or a straight link if the the two joint axes do not intersect.

From Fig. 1, We obtain

$$|AB| = 2|CA| \sin(\angle ACB/2) = 2|DA| \sin(\angle ADB/2) \quad (1)$$

It is noted that  $\angle ACB = \angle ACB_1 + \angle B_1CB = \angle A_1CA + \angle ACB_1 = \angle A_1CB_1$ ,  $|CA_1| = |CB_1|$ ,  $\angle ADB = \angle ADA_2 + \angle A_2DB = \angle A_2DB + \angle BDB_2 = \angle A_2DB_2$ ,  $|DA_2| = |DB_2|$ ,  $|CA| = \sqrt{|CA_1|^2 + r^2}$  and  $|DA| = \sqrt{|DA_2|^2 + r^2}$ .

Eq. (1) then becomes

$$\sqrt{|CA_1|^2 + r^2} \sin(\angle A_1CB_1/2) = \sqrt{|DA_2|^2 + r^2} \sin(\angle A_2DB_2/2) \quad (2)$$

## 2.2. Link parameters of the 6R mechanism

Different variations of D-H notations are used in the literature [8, 12, 14, 31, 32, 33]. The coordinate frames are attached to the links and the link parameters are defined as in [33] (Fig. 2).  $Z_i$ -axis is along the axis of joint  $i$ .  $X_i$ -axis is along the common perpendicular between  $Z_{i-1}$ - and  $Z_i$ -axes.  $O_i$  is the intersection of  $X_i$ - and  $Z_i$ -axes.  $Y_i$ -axis is defined by  $X_i$ - and  $Z_i$ -axes through the right handed rule. The link parameters of link  $i$  are:  $d_i$  (the distance between  $X_i$ - and  $X_{i+1}$ -axes measured from  $X_i$ -axis to  $X_{i+1}$ -axis along  $Z_i$ -axis),  $\alpha_i$  (the twist angle between  $Z_i$ - and  $Z_{i+1}$ -axes measured from  $Z_i$ -axis to  $Z_{i+1}$ -axis about  $X_{i+1}$ -axis), and  $l_i$  (the distance between  $Z_i$ - and  $Z_{i+1}$ -axes measured from  $Z_i$ -axis to  $Z_{i+1}$ -axis along  $X_{i+1}$ -axis). Throughout

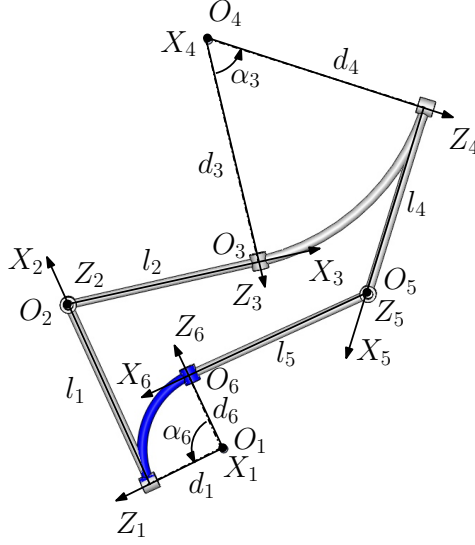


Figure 2: D-H parameters of the 6R mechanism.

this paper, link 6, which connects joints 6 and 1, is selected as the frame of the 6R mechanism and highlighted in blue.

The link parameters of the 6R mechanism are:  $\alpha_1 = \alpha_2 = \alpha_4 = \alpha_5 = \pi/2$ ,  $\alpha_3, \alpha_6, d_4 = -d_3, d_2 = d_5 = 0, d_6 = -d_1, l_1 = l_2 = l_4 = l_5, l_3 = 0$ , and  $l_6 = 0$ .

From the construction of the 6R mechanism (Fig. 1), we learn that the link parameters of the 6R mechanism satisfy the following constraint (see Eq. (2)) :

$$\sqrt{d_3^2 + l_1^2} |\sin(\alpha_3/2)| = \sqrt{d_1^2 + l_1^2} |\sin(\alpha_6/2)| \quad (3)$$

As pointed out by one of the reviewers of this paper, this 6R mechanism that has three pairs of R joints with intersecting joint axes is a special case of the Wohlhart 6R mechanism [8] obtained by merging two Goldberg 5R mechanisms. The contribution in this paper is that this 6R mechanism is obtained by a concise geometric construction. If the kite for constructing the 6R mechanism (Fig. 1) is turned into a rhombus, then the 6R mechanism that has three pairs of R joints with intersecting joint axes is a special case of a Bricard line symmetric 6R mechanism.

### 3. Kinematic Analysis of the 6R mechanism that has three pairs of R joints with intersecting joint axes

In this section, we will present the kinematic analysis of the 6R mechanism constructed in Section 2. Since the approaches to the kinematic analysis of 6R mechanisms have been well documented in the literature (see [6, 7, 12, 14, 20, 31, 32, 34, 35] for example), detailed derivation will be omitted in this paper. The method for kinematic analysis in [8] is not used in this paper since it requires one to first determine the link parameters of two 5R Goldberg mechanisms and then perform the kinematic analysis of these two 5R Goldberg mechanisms.

The link parameters of an example mechanism — Mechanism I (Fig. 2) — are given as follows:  $l_1 = l_2 = l_4 = l_5 = 120$ ,  $l_3 = 0$ ,  $l_6 = 0$ ,  $\alpha_1 = \alpha_2 = \alpha_4 = \alpha_5 = \pi/2$ ,  $\alpha_3 = \pi/3$ ,  $\alpha_6 = \pi/2$ ,  $d_2 = d_5 = 0$ , and  $d_1 = -d_6 = 50$ .

Using Eq. (3), we obtain the link parameters  $d_4$  and  $d_3$  as  $d_4 = -d_3 = 10\sqrt{194}$ .

The input-output equation of Mechanism I is derived as

$$\begin{aligned} & -169S\theta_1 S\theta_2 \sqrt{194} + 2028S\theta_1 C\theta_2 + 119\sqrt{194}C\theta_2 \\ & + 120S\theta_2 \sqrt{194} + 2028S\theta_1 - 1440C\theta_2 + 1428S\theta_2 - 845 = 0 \end{aligned} \quad (4)$$

For a given set of  $\theta_1$  and  $\theta_2$ , one can determine  $\theta_i$  ( $i = 3, 4, \dots, 6$ ) by calculating  $t_i = \cot(\theta_i/2)$  using the following equations.

$$t_3 = -u_2/u_1 \quad (5)$$

where

$$u_1 = 20111t_1^2 + (4056\sqrt{194}\sqrt{3} + 60840)t_1 - 1690\sqrt{194}\sqrt{3} - 45461$$

$$\begin{aligned} u_2 = & (595\sqrt{194}\sqrt{3} - 11425\sqrt{3} - 2285\sqrt{194} + 23086)t_1^2 t_2 \\ & + (2628\sqrt{194}\sqrt{3} + 7140\sqrt{3} + 1428\sqrt{194} + 53700)t_1^2 \\ & + (40560\sqrt{3} + 8112\sqrt{194})t_1 t_2 \\ & + (-65572 - 1690\sqrt{194}\sqrt{3})t_1 \\ & + (595\sqrt{194}\sqrt{3} - 11425\sqrt{3} - 2285\sqrt{194} + 23086)t_2 \\ & - 1428\sqrt{194}\sqrt{3} + 1428\sqrt{194} + 7140\sqrt{3} - 7140 \end{aligned}$$

$$t_4 = (\sqrt{194}\sqrt{3}t_3 + 36)/(\sqrt{194}\sqrt{3} + 36t_3) \quad (6)$$

$$t_5 = (\sqrt{194}t_2 + 10t_2 - 12)/(12t_2 + \sqrt{194} + 10) \quad (7)$$

$$t_6 = -(5t_1 - 12)/(12t_1 - 5) \quad (8)$$

The variation of  $\theta_i$  ( $i=2, 3, \dots, 6$ ) with respect to  $\theta_1$  for Mechanism I is shown in Fig. 3. It can be readily verified using Eq. (6) that the axes of joints 2 and 5 are always coplanar during the motion of the 6R mechanism. It is observed that Mechanism I has two circuits with joints 1, 6, 3 and 4 as full-turn R joints. Figures 4 and 5 show Mechanism I at configurations A, B and C in circuit 1 and configurations D, E and F in circuit 2 respectively. It is observed that at two configurations in each circuit (see configurations A and C in circuit 1 and configurations D and F in circuit 2), the axes of R joints 1, 3, 4 and 6 of Mechanism I are coplanar and the axes of R joints 2 and 5 are perpendicular to the plane defined by the axes of R joints 1, 3, 4 and 6.

#### 4. Discussion

The above results have been verified using several mechanism models built using 3D printing. Figure 6 shows the CAD model and 3D-printed prototype of 6R Mechanism I (Fig.4c). It is noted that joints 1 and 6 in this prototype are prevented from full-cycle rotation due to interference between links 2 and 4 as well as links 1 and 5.

Let  $K_1$ ,  $K_2$  and  $K_3$  denote the intersections of joint axes of joints 1 and 6, joints 2 and 5, and joints 3 and 4.  $\Pi_1$ ,  $\Pi_2$  and  $\Pi_3$  represent the plane defined by the axes of joints 1 and 6, joints 2 and 5, and joints 3 and 4 respectively (Fig. 7). From [36], we obtain that planes  $\Pi_1, \Pi_2$  and  $\Pi_3$  and plane  $K_1K_2K_3$  have a common point,  $K$ , at any configuration of the 6R mechanism during motion. It is noted that at configurations A, C (Fig. 4), D and F (Fig. 5) of Mechanism I, point  $K_2$  is at infinity.

#### 5. Conclusions

A 6R mechanism that has three pairs of R joints with intersecting joint axes has been proposed using a geometric construction approach. Kinematic analysis of the mechanism has been presented. The analysis has shown that the 6R mechanism usually has two solutions to the kinematic analysis for a given input. In two configurations in each circuit of the 6R mechanism, the



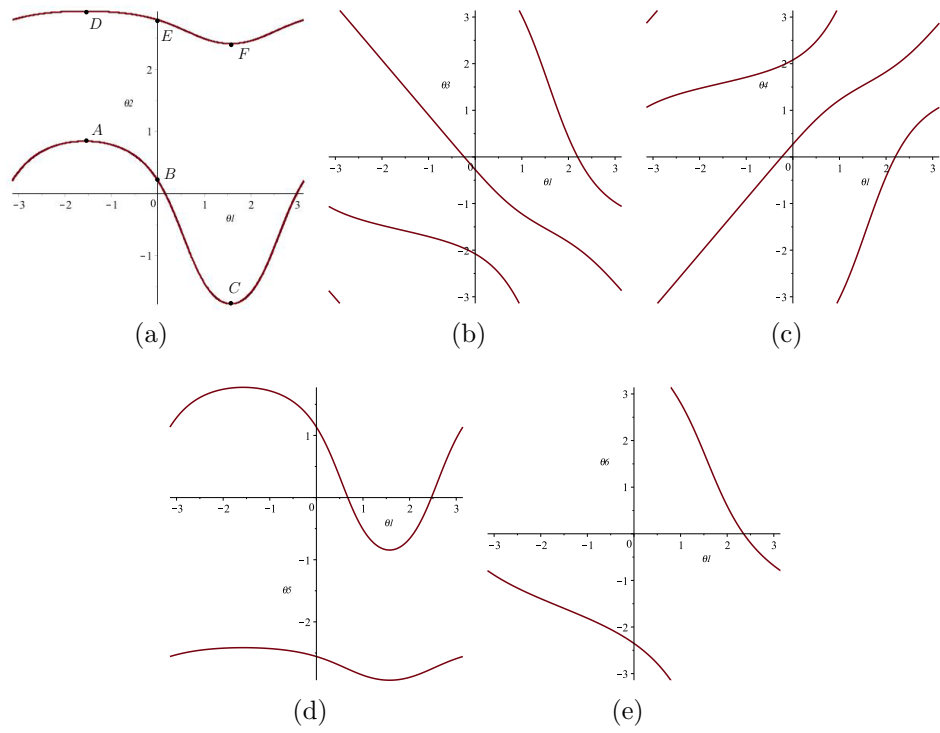


Figure 3: Kinematic analysis of Mechanism I: (a) Plot of  $\theta_1 - \theta_2$ , (b) Plot of  $\theta_1 - \theta_3$ , (c) Plot of  $\theta_1 - \theta_4$ , (d) Plot of  $\theta_1 - \theta_5$ , (e) Plot of  $\theta_1 - \theta_6$ .

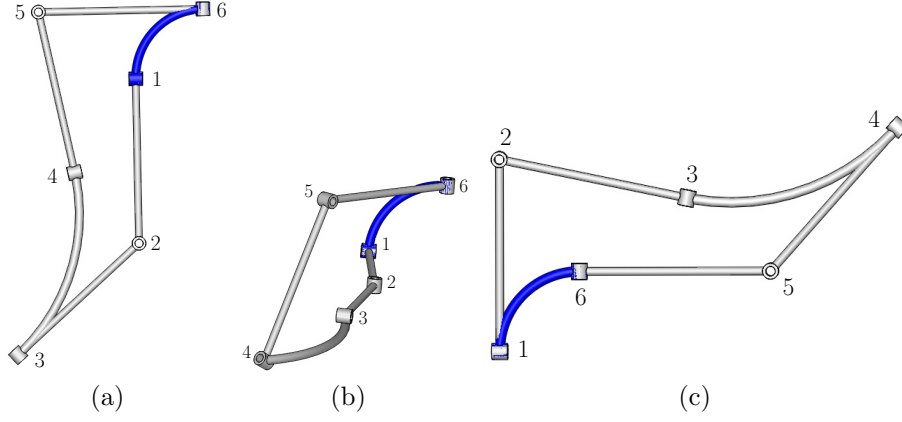


Figure 4: Configurations of Mechanism I in Circuit 1: (a) Configuration A:  $\theta_1 = -\pi/2$ , (b) Configuration B:  $\theta_1 = 0$ , and (c) Configuration C:  $\theta_1 = \pi/2$ .

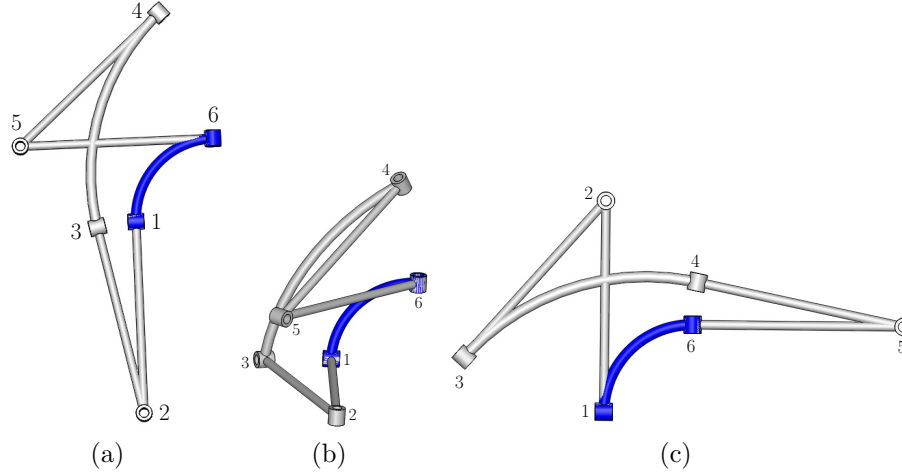


Figure 5: Configurations of Mechanism I in Circuit 2: (a) Configuration D:  $\theta_1 = -\pi/2$ , (b) Configuration E:  $\theta_1 = 0$ , and (c) Configuration F:  $\theta_1 = \pi/2$ .

axes of four R joints are coplanar, and the axes of the other two R joints are perpendicular to the plane defined by the above four R joints.

Although the 6R mechanism that has three pairs of R joints with intersecting joint axes proposed in this paper is a special case of Wohlhart's 6R mechanism based on two Goldberg 5R mechanisms, it is obtained here using a very concise geometric construction. The kinematic analysis of the 6R mechanism in this paper is also more concise than that in reference [8], where

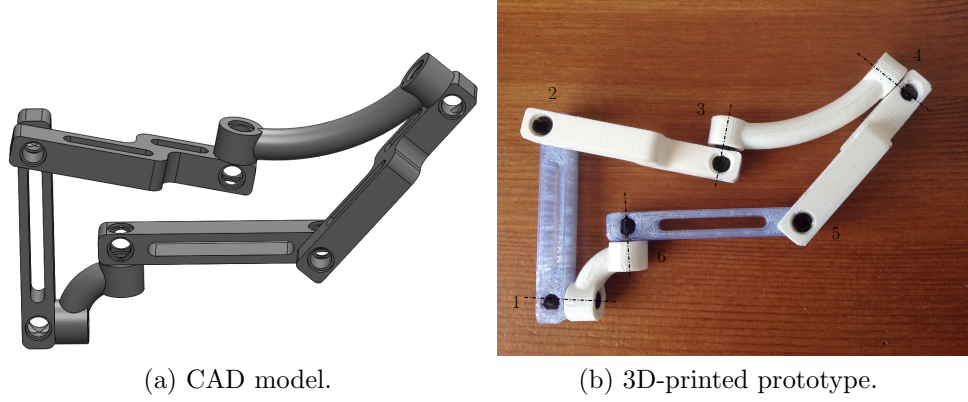


Figure 6: A prototype of 6R Mechanism I.

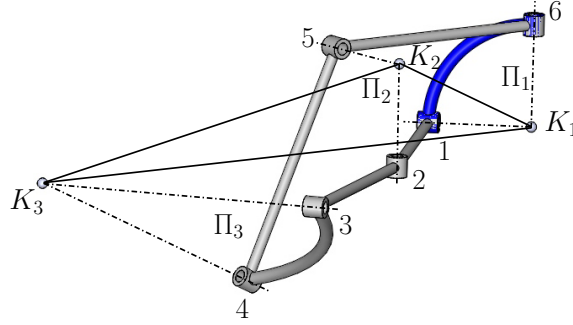


Figure 7: Characteristics of the 6R mechanism.

the kinematic analysis was based on the kinematic analysis of two Goldberg 5R mechanisms.

This work enriches the geometric approach for identifying 6R overconstrained mechanisms and is a step forward in the research on 6R mechanisms that have three pairs of R joints with intersecting joint axes.

### Acknowledgment

The author would like to thank the Engineering and Physical Sciences Research Council (EPSRC), United Kingdom, for the support under grant No. EP/K031643/1.

## References

- [1] R. Bricard, Mémoire sur la théorie de l'octaèdre articulé, *Journal de Mathématiques Pures et Appliquées* 3 (1897) 113–148.
- [2] R. Bricard, *Lecons de cinématique: Tome II Cinématique Appliquée*, Gauthier-Villars, Paris, 1927.
- [3] M. Goldberg, New five-bar and six-bar linkages in three dimensions, *Transactions of the ASME* 65 (1943) 649–663.
- [4] K. Waldron, Hybrid overconstrained linkages, *Journal of Mechanisms* 2 (1968) 73–78.
- [5] K. Wohlhart, A new 6R space mechanism, in: *Proceedings of the 7th World Congress on Theory of machines and mechanisms*, Seville, Spain, 1987, pp. 193–198.
- [6] C. Mavroidis, B. Roth, Analysis of overconstrained mechanism, *ASME Journal of Mechanical Design* 117 (1995) 69–74.
- [7] P. Dietmaier, A new 6R space mechanism, in: *Proceedings of the 9th World Congress IFToMM*, Milano, Italy, 1995, pp. 52–56.
- [8] K. Wohlhart, Merging two general goldberg 5R linkages to obtain a new 6R space mechanism, *Mechanism and Machine Theory* 26 (2) (1991) 659–668.
- [9] K. Six, A. Kecskeméthy, Steering properties of a combined wheeled and legged striding excavator, in: *Proceedings of the 10th World Congress on the Theory of Machines and Mechanisms*, Oulu, Finland, 1999, pp. 135–140.
- [10] P. Zsombor-Murray, A. Gferrer, 'robotrac' mobile 6R closed chain, in: *Proceedings of CSME Forum 2002*, Kingston, Canada, 2002, pp. 02–05.
- [11] J. Baker, Overconstrained six-bars with parallel adjacent joint-axes, *Mechanism and Machine Theory* 38 (2) (2003) 103–117.
- [12] J. Baker, A curious new family of overconstrained six-bars, *ASME Journal of Mechanical Design* 127 (4) (2005) 602–606.

- [13] J. Baker, On generating a class of foldable six-bar spatial linkages, *ASME Journal of Mechanical Design* 128 (2) (2006) 374–383.
- [14] Y. Chen, Z. You, Spatial 6R linkages based on the combination of two goldberg 5R linkages, *Mechanism Machine Theory* 42 (2007) 1484–1498.
- [15] J. Baker, Screw replacements in isomeric variants of bricard’s line-symmetric six-bar, *Proceedings of the Institution of Mechanical Engineers. Part C: Journal of Mechanical Engineering Science* 223 (10) (2009) 2391–2398.
- [16] M. Pfullner, A new family of overconstrained 6R-mechanisms, in: M. Ceccarelli (Ed.), *Proceedings of EUCOMES 08*, Springer, The Netherlands, 2009, pp. 117–124.
- [17] G. Hegedüs, J. Schicho, H.-P. Schröcker, Construction of overconstrained linkages by factorization of rational motions, in: J. Lenarčič, M. Husty (Eds.), *Latest Advances in Robot Kinematics*, Springer, The Netherlands, 2012, pp. 213–220.
- [18] Z. Li, J. Schicho, Classification of angle-symmetric 6R linkages, *Mechanism and Machine Theory* 70 (2013) 372–379.
- [19] Z. Li, J. Schicho, Three types of parallel 6R linkages, in: F. T. A., P. Gracia (Eds.), *Computational Kinematics*, Springer, The Netherlands, 2014, pp. 111–119.
- [20] Z. Li, Sharp linkages, in: J. Lenarcic, O. Khatib (Eds.), *Advances in Robot Kinematics*, Springer, The Netherlands, 2014, pp. 131–138.
- [21] G. Hegedüs, Z. Li, J. Schicho, H.-P. Schröcker, The theory of bonds ii: Closed 6R linkages with maximal genus, *Journal of Symbolic Computation* 68 (2) (2015) 167–180.
- [22] X. Kong, Type synthesis of single-loop overconstrained 6R mechanisms for circular translation, *Journal of Mechanisms and Robotics* 6 (4) (2014) 041016.
- [23] Z. Li, J. Schicho, A technique for deriving equational conditions on the denavit-hartenberg parameters of 6R linkages that are necessary for movability, *Mechanism and Machine Theory* 94 (2015) 1–8.

- [24] C. Lee, J. Hervé, Geometric derivation of 6R linkages with circular translation, in: J. Lenarčič, O. Khatib (Eds.), *Advances in Robot Kinematics*, Springer, The Netherlands, 2014, pp. 59–67.
- [25] M. Yarullin, M. R. Mingazov, I. Galiullin, Historical review of studies of spatial nR linkages, in: *Proceedings of 2015 IFToMM Workshop on History of Mechanism and Machine Science*, St-Petersburg, Russia, 2015, p. 6 pages.
- [26] M. L. Husty, H.-P. Schröcker, A proposal for a new definition of the degree of freedom of a mechanism, in: A. Kecskeméthy, V. Potkonjak, A. Müller (Eds.), *Interdisciplinary Applications of Kinematics*, Springer, The Netherlands, 2008, pp. 109–117.
- [27] X. Kong, C. Gosselin, *Type Synthesis of Parallel Mechanisms*, Springer, The Netherlands, 2007.
- [28] X. Kong, C. Gosselin, Type synthesis of three-degree-of-freedom spherical parallel manipulators, *International Journal of Robotics Research* 23 (3) (2004) 237–245.
- [29] C. Liu, S. Yao, H. Wang, Y. Yao, Mobile schatz mechanism, *Journal of Mechanisms and Robotics* 8 (1) (2015) 015001.
- [30] L. Racila, M. Dahan, Spatial properties of wohlhart symmetric mechanism, *Meccanica* 45 (2010) 153–165.
- [31] C.-C. Lee, H.-S. Yan, Movable spatial 6R mechanisms with three adjacent parallel axes, *ASME Journal of Mechanical Design* 115 (3) (1993) 522–529.
- [32] Q. Jin, T.-L. Yang, Overconstraint analysis on spatial 6-link loops, *Mechanism and Machine Theory* 37 (3) (2002) 267–278.
- [33] X. Kong, Kinematic analysis of a 6R single-loop overconstrained spatial mechanism for circular translation, *Mech Mach Theory* 93 (2015) 163–174.
- [34] J. Baker, Displacement-closure equations of the unspecialised double-hookes-joint linkage, *Mechanism and Machine Theory* 37 (10) (2002) 1127–1144.

- [35] M. Pfurner, M. Husty, A method to determine the motion of overconstrained 6R-mechanisms, in: Proceedings of the 12th IFToMM World Congress in Mechanism and Machine Science, Besancon, France, 2007, pp. 718–725.
- [36] X. Kong, A. Johnson, Classification of screw systems composed of three planar pencils of lines for singularity analysis of parallel mechanisms, *Journal of Mechanisms and Robotics* 6 (2) (2014) 021008.